Math 60

Chapter 10: Quadratic Equations and Quadratic Functions

10.1	Solving quadratic equations by completing the square	(1 day)
10.2	Solving quadratic equations by the quadratic formula	(1 day)
10.3	Solving equations that are quadratic in form	(2 days)
10.4	Graphing quadratic functions using transformations	(1 day)
10.5	Graphing quadratic functions using properties	(1 day)
10.6	Polynomial Inequalities	(2 days)

{not 10.7}

Math 60 10.1 Solving Quadratic Equations by Completing The Square

Objectives 1) Solve quadratic equations using the square root property
2) Complete the square in one variable
3) Solve quadratic equations by completing the square.
4) Solve problems using the Pythagorean Theorem.

Review:

① Solve $x^2 - 5x = -6$.

Step 1: Notice x^2, which makes this a quadratic eqn.

Step 2: Set $= 0$.

$$x^2 - 5x + 6 = 0$$

Step 3: Factor completely

$$(x-2)(x-3) = 0$$

Step 4: Set factors $= 0$ and isolate the variable

$$x - 2 = 0 \qquad x - 3 = 0$$

$$\boxed{x = 2 \qquad x = 3}$$

But what if the trinomial does not factor?

② $x^2 - 5x + 2 = 0$

GOAL: Get another method for finding values of x that make this equation true.

Actually, we'll get two methods
1. Solve by completing the square (today)
2. Solve by the quadratic formula (next class).

Let's build up to this.

(3) Solve $x^2-9=0$ by factoring

$(x-3)(x+3)=0$ factor

$x-3=0 \quad x+3=0$ factors $=0$

$\boxed{x=3, x=-3}$ isolate

Notice: The two solutions can be written as

$\boxed{x=\pm 3}$

(4) Solve $x^2-9=0$ using the square root property

Step 1: Isolate the square

$x^2=9$

Step 2: Use the square root property. This says we are allowed to take the square root of both sides if we remember..... $\sqrt{x^2}=|x|$

Short cut: use $\pm$

$\sqrt{x^2}=\sqrt{9}$

$|x|=3$

$\boxed{x=\pm 3}$

$\sqrt{x^2}=\pm\sqrt{9}$

$\boxed{x=\pm 3}$

(5) Solve $\underbrace{(x-2)^2}=9$ by the square root property

Step 1 Recognize: This is a perfect square. It is isolated!

Step 2: Use square root property

$\sqrt{(x-2)^2}=\sqrt{9}$

$x-2=\pm 3$

Step 3: write two equations $x-2=3 \quad x-2=-3$

$\boxed{x=5 \qquad x=-1}$

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6 Solve $(2x-3)^2 = 7$.

$$\sqrt{(2x-3)^2} = \pm\sqrt{7}$$ square root property

$2x-3 = +\sqrt{7}$ $\quad$ $2x-3 = -\sqrt{7}$ $\quad$ 2 eqns.

$2x = 3+\sqrt{7}$ $\quad$ $2x = 3-\sqrt{7}$

$\boxed{x = \frac{3}{2} + \frac{\sqrt{7}}{2}}$ $\quad$ $\boxed{x = \frac{3}{2} - \frac{\sqrt{7}}{2}}$

This answer can also be written using $\pm$

$$\boxed{x = \frac{3}{2} \pm \frac{\sqrt{7}}{2}}$$

It can also be written as one fraction

$\boxed{x = \frac{3 \pm \sqrt{7}}{2}}$ ← However this can lead to bad habits! See next example.

7 Solve $(2x-3)^2 = -4$

$2x-3 = \pm\sqrt{-4}$ $\quad$ square root property

$2x-3 = \pm 2i$ $\quad$ simplify imaginary #

$2x-3 = 2i$ $\quad$ $2x-3 = -2i$

$\frac{2x}{2} = \frac{3}{2} + \frac{2i}{2}$ $\quad$ $\frac{2x}{2} = \frac{3}{2} - \frac{2i}{2}$

$\boxed{x = \frac{3}{2} + i}$ $\quad$ $\boxed{x = \frac{3}{2} - i}$

We should always write complex number answers in standard form, $a+bi$ (not a big fraction).

$\boxed{x = \frac{3}{2} \pm i}$ is also acceptable.

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But what if it's not a perfect square?
Then we build a perfect square.

Review: 8 Simplify $(x+3)^2$

$$= (x+3)(x+3)$$
$$= x^2 + 3x + 3x + 9$$
$$= x^2 + 2\cdot 3x \quad 9$$
$$= \boxed{x^2 + 6x + 9}$$

perfect square trinomial

GOAL: Starting from $6x$,
find a) the number 9
b) the expression $x+3$.

(CTS)
9 Complete the square $x^2 + 6x = 2$

step 1: make sure its $1x^2$, not $2x^2$ or $-x^2$ or any other coefficient besides 1.

CAUTION: CTS will NOT work (not ever) if the leading coefficient is not 1.

step 2: Take half of the coefficient of the middle term. $6x$

$\Rightarrow 6$

$\Rightarrow \frac{1}{2}\cdot 6$ or $\frac{6}{2}$

Hint: Sometimes mult by $\frac{1}{2}$ is easier.

$\Rightarrow 3.$ This is the number we need for the expression $(x+3)$.

step 3: Square the result

$3^2 = 9$ This is the number we need to add to build a perfect square.

cont $\Rightarrow$

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step 4: In the original eq'n, add the squared number to both sides of the = sign

$$x^2+6x+9 = 2+9$$

step 5: Recall that 3 would be used to find $(x+3)$. Write the LHS as a perfect square + add RHS.

$$\boxed{(x+3)^2 = 11}$$

We have finished the process of "completing the square" (CTS) ⇒ meaning that we found a number we could add to both sides to make a perfect square trinomial.

We could now solve the equation.

(10) Solve $(x+3)^2 = 11$

$$x+3 = \pm\sqrt{11}$$

$$\boxed{x = -3\pm\sqrt{11}}$$

(11) Solve $3x^2-60=0$

step 1: Isolate the square

$$\frac{3x^2}{3} = \frac{60}{3}$$

$$x^2 = 20$$

step 2: Square root property

$$x = \pm\sqrt{20}$$

$$x = \pm\sqrt{4\cdot 5}$$

$$\boxed{x = \pm 2\sqrt{5}}$$

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"BUT... I learned the quadratic formula before, and I can do that better than completing the square.

WHY SHOULD I BOTHER LEARNING CTS?"

Because you will need CTS in 12.2 when we work on equations of circles, and there's no "circle formula" to save you (as there was a "vertex formula" to save you with parabolas).

STEM majors: You may need CTS to solve certain kinds of quadratic equations for one of two or more variables.

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Solve by CTS.

(12) $x^2-8x+9=0$

step 1: check x^2 has coef 1. ✓
move the existing 9 to RHS

$x^2-8. \quad = -9$

step 2: CTS $\quad \frac{-8}{2} = -4 \Leftarrow$ # needed for factor

$(-4)^2 = 16 \Leftarrow$ # added to both sides

$x^2-8x+16 = -9+16$

step 3: write perfect square, add RHS

$(x-4)^2 = 7$

step 4: solve by square root property

$x-4 = \pm\sqrt{7}$

$\boxed{x = 4 \pm \sqrt{7}}$

(13) Solve $2x^2+4x+3=0$

step 1: $2x^2$!! Must divide all terms by 2.

$\frac{2x^2}{2} + \frac{4x}{2} + \frac{3}{2} = \frac{0}{2}$

$x^2 + 2x + \frac{3}{2} = 0$

step 2: move $\frac{3}{2}$ to RHS

$x^2+2x \quad = -\frac{3}{2}$

step 3: CTS $\quad \frac{2}{2} = 1 \Leftarrow$ # for factor

$1^2 = 1 \Leftarrow$ # to add both sides

$x^2+2x+1 = -\frac{3}{2}+1$

cont $\Rightarrow$

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$(x+1)^2 = -\frac{1}{2}$ write perfect square

$x+1 = \pm\sqrt{\frac{-1}{2}}$ square root property

$x = -1 \pm \frac{\sqrt{-1}}{\sqrt{2}}$ isolate x

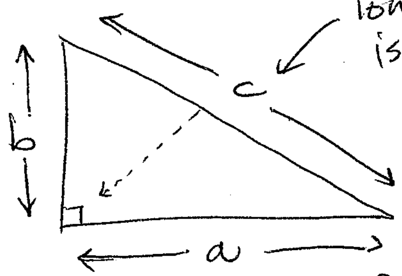
$= -1 \pm \frac{i}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ simplify $\sqrt{\ }$
rationalize denom.

$= \boxed{-1 \pm \frac{i\sqrt{2}}{2}}$

Pythagorean Theorem

Right Triangle

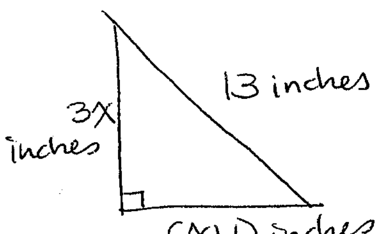
$a^2+b^2=c^2$



longest side, hypotenuse, is across from the right angle

* CAUTION *
When using algebraic expressions for side lengths, be sure to use parentheses!

(14) Find the sides of the right triangle



$a^2+b^2=c^2$

$(x+1)^2+(3x)^2=13^2$ subst

This has x in it – can't use square property yet!

$(x+1)(x+1) + 9x^2 = 169$

$x^2+2x+1 + 9x^2 = 169$ FOIL

$\frac{10x^2}{10} + \frac{2x}{10} = \frac{168}{10}$ divide to get x^2

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$x^2 + \frac{1}{5}x = \frac{84}{5}$ reduce fractions

↑

CTS take half $\frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$

Remember: $\div 2$ is the same as mult by $\frac{1}{2}$

square $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$

$\underbrace{x^2 + \frac{1}{5}x + \frac{1}{100}} = \frac{84}{5} + \frac{1}{100}$ add to both sides

use work above to write factor!

$\left(x + \frac{1}{10}\right)^2 = \frac{84 \cdot 20}{5 \cdot 20} + \frac{1}{100}$ find CD

$\left(x + \frac{1}{10}\right)^2 = \frac{1680 + 1}{100}$ add fractions.

$\left(x + \frac{1}{10}\right)^2 = \frac{1681}{100}$

$x + \frac{1}{10} = \pm \frac{\sqrt{1681}}{\sqrt{100}}$

$x + \frac{1}{10} = \pm \frac{41}{10}$

$x = \frac{-1}{10} \pm \frac{41}{10}$

$x = \frac{40}{10}$ or $\left(\frac{-1}{10} \frac{-41}{10} = \frac{-42}{10} = \frac{-21}{5} \right.$

$x = 4$ extraneous - can't have negative length $\left. \right)$

Sides of triangle

$3x \Rightarrow 3 \cdot 4 =$ 12 inches

$x+1 \Rightarrow 4+1 =$ 5 inches

given 13 inches